The Preparation of Secondary Pre- and Inservice Mathematics Teachers on the Integration of Technology in Topics Foundational to Calculus

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Abstract

Because the graphing calculator has been on the market for 25 years, it is fair to ask whether we are taking full advantage of the capabilities that hand-held graphing technology (HHGT) offers, without Computer Algebra Systems (CAS). In particular, are we using HHGT to provide students with the best possible preparation for calculus? To answer this question in the USA, we first established criteria on how the integration of technology expands the study of families of continuous functions at this level. Then, we proceeded to explore the knowledge on the integration of HHGT that secondary pre- and inservice teachers have. A test based on the established criteria was administered to three intact groups consisting of 46 preservice secondary teachers from three universities in the Midwest of the USA. The same test was also given to a group of 74 secondary inservice teachers representing 40 school districts from the same geographical area. The test results were very low. The teachers were also asked to answer a survey where they rated their knowledge on the established criteria, immediately before taking the test. The self-evaluation of both groups on the chosen topics ranged from "very little" to "some" knowledge, corroborating their self-awareness on their lack of preparation on these topics. This issue does not appear to be limited to precalculus topics. Another group of inservice teachers were tested over using HHGT in probability, statistics, data analysis, matrices and discrete mathematics. The results in this case were even lower than for the precalculus topics.

1. Introduction

The integration of technology into the teaching and learning of mathematics impacts every aspect of instruction, from course content to teaching methods and assessment. As a result, some of the assumptions made about mathematics curricula prior to the integration of technology in the classroom are no longer valid. The ability to bridge cumbersome calculations via technology allows students at various levels to meaningfully explore concepts and problems that were previously proposed only to more advanced mathematics students and allows teachers to extend the breadth and depth while presenting these concepts. Therefore topics such as optimization, regression, recursion and others are now accessible to secondary students at different levels prior to calculus instruction. More importantly, the numerical and graphical capabilities of hand-held graphing technology (HHGT) allows the introduction of key concepts foundational to calculus through the use of approximations; resembling a way that is similar to how these concepts were developed and we believe, can be more easily understood.

The National Council of Teachers of Mathematics (NCTM) [6] *Curriculum and Evaluation Standards for School Mathematics* recommends that:

> In grades 9-12, the mathematics curriculum should include the informal exploration of calculus concepts from both a graphical and a numerical perspective so that all students can -
>
> - Determine maximum and minimum points of a graph and interpret the results in problem situations;
> - Investigate limiting processes by examining infinite sequences and series and areas under curves;
>
> and so that, in addition, college-intending students can-
>
> - Understand the conceptual foundations of limit; the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines;
> - Analyze the graphs of polynomial, rational, radical, and transcendental functions. (p. 180).

According to Jockusch and McLoughlin [5] “These concepts can be developed as natural extensions of topics that students have already encountered” (p. 532). Orton [8] maintains that the groundwork for understanding calculus can be laid for many students, before the age of 16, through exploration using lower level skills and that calculators can be used to facilitate these explorations. Stroup [13] comments,

> Traditionally, we think of calculus as a culminating course in a secondary mathematics curriculum. It seems backward then, to suggest that calculus can help our younger students make better sense of topics we typically label “the basics”. If anything, we assume in our curricula and in our teaching that calculus is a subject to be studied well after the basics are mastered and only after a long series of prerequisite coursework has been taken. As a result, most of our students do not progress as far as calculus; this limits them in terms of their opportunities in post-secondary education. It also limits them in terms of the formal mathematical tools they can bring to situations where rate varies. The world in which we live is dynamic and changing, and all our students should develop powerful ways of talking about change. (p.180)

The NCTM [7] technology principle for school mathematics states that “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (p. 24). They elaborate that “The existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it” (p. 25). Hughes-Hallett [4] backs up these statements by pointing out that “computers and calculators can now easily compute definite integrals, sketch graphs, solve equations, and find high powers of matrices” (p. 1547) and further calculators can “… do the algebraic manipulations that have been the backbone of high school mathematics for decades” (p. 1547). Thus leading Hughes-Hallett to conclude that we need to reconsider what we are teaching and how we are teaching.

Given then the importance that calculus and, indeed, concepts foundational to calculus have, and the role that technology may play in enhancing and expanding these concepts, we believe that 25 years after hand-held graphing technology (HHGT) came to the market, it is appropriate to ask: *Are teachers taking full advantage of the main capabilities that this technology offers in order to provide precalculus students with the best possible preparation for calculus?* Based on our experience with college students in a variety of undergraduate mathematics classes, we expected to find evidence that secondary teachers were not taking full advantage of what HHGT could offer.

We are not making any reference to HHGT with CAS because, in our view, we should agree on the proper use of HHGT without CAS before we address the new set of curricular and pedagogical questions that symbolic calculators would bring to the schools. We remark that the term precalculus is used throughout the paper to indicate not a particular course, but rather courses before calculus.

To answer the aforementioned question in the USA, we first decided to establish criteria postulating how the integration of technology might expand the study of families of continuous functions at this level. Then, we developed a test containing mostly conceptual questions for evaluating the comprehension of the concepts addressed in these criteria.

2. How the integration of technology impacts and expands the study of functions in precalculus

We have previously proposed [10] that technology can be utilized in the classroom to:

1. Increase the emphasis on conceptual understanding and exploration [3] [12].
2. Introduce relevant concepts and applications now accessible at this level [9].
3. Facilitate the continuous interplay of the graphical, numerical, and analytical representations with every family of continuous functions.
4. Provide a uniform approach to the study of each family of continuous functions through transformations such as $\boldsymbol{f(x)+a}$, $\boldsymbol{f(x+a)}$, $\boldsymbol{-f(x)}$, $\boldsymbol{a\cdot f(x)}$, $\boldsymbol{f(ax)}$, $\boldsymbol{|f(x)|}$, and $\boldsymbol{f(|x|)}$.
5. Provide new approaches to problem solving via the newly available data types.

The simultaneous introduction of the graphical, numerical and analytical aspects of functions makes it possible to use Demana and Waits [1] approach of having students support: i) analytical solutions graphically and/or numerically, and ii) graphical and numerical solutions analytically, whenever possible.

In addition to the properties traditionally considered for every family of continuous functions, the integration of HHGT allows one to approximate irrational zeros as well as local extrema; properties that are not always available analytically at this level. Hence, it is feasible to extend the study of continuous functions at the precalculus level by adding the following topics:

1. Finding the range of continuous functions studied,
2. Determining irrational zeros, hence all real zeros,
3. Finding local extrema, hence intervals where functions are increasing or decreasing.

Moreover, HHGT (with or without CAS) provides data types such as tables, lists, sequences, and matrices. They also support the use of regression, recursive functions, and the ability to use recursion from the home screen [9]. These resources allow further expansion of the traditional approach to the study of families of continuous functions through:

1. Using sequences to explore local and end behavior,
2. Comparing relative growth of functions from the same or different families,
3. Using real world connections by considering relevant examples of data that can be modeled, via regression by the family of functions studied,
4. Including optimization problems.

Finally, the latest models of HHGT have incorporated two new environments: a spreadsheet and dynamic geometry software, with the capability of recognizing any user-defined variable in any subsequent environment. Since technology enables students to revisit problems from different perspectives, it is possible to use a spiral approach with some of these concepts, like optimization, through different courses preceding calculus by solving problems using the concepts and tools accessible to their particular level [11].

These are ways that technology integration can be used to alter how and what we teach in mathematics' classes prerequisite to calculus. Before describing the methods we used to determine whether teachers are taking advantage of these ways to integrate technology when teaching precalculus mathematics, examples that demonstrate this integration will be illustrated.

3. Some examples on new concepts and approaches fostered by HHGT

In this section we present several examples to illustrate both new concepts now accessible to precalculus students, and new approaches for solving these problems. We are not trying to exhaust all of the accessible new concepts or approaches, but rather to facilitate understanding of the following sections in this article. In each example we include some figures obtained using the TI-Nspire calculator, and briefly point to the different ways in which the solution is found.

Example 1. Solving equations and inequalities graphically.

Solve a) $f1(x) = f2(x)$ for $f1(x) = -5x^2 + 6$ and $f2(x) = -x^3 + 3x^2 + x + 1$;

b) $f3(x) = f4(x)$ and $f3(x) < f4(x)$ for $f3(x) = 10x^2$ and $f4(x) = 0.1e^x$.

This first example demonstrates how graphical and tabular representations can be combined to find the intersections of two continuous functions while learning about the relative growth of the functions. This method can be used before introducing students to other strategies for solving these problems (such as factoring or the fundamental theorem of algebra). Figure 1 shows the default

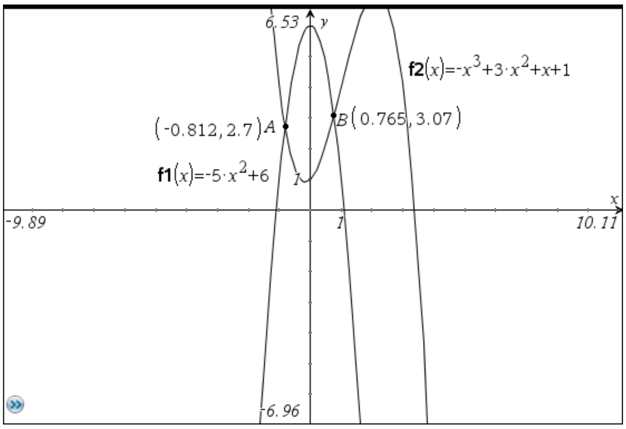


Figure 1: *Default window for 1 a)*

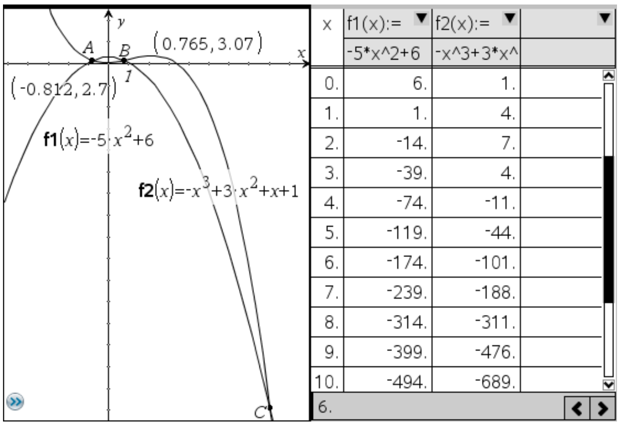


Figure 2: *Improved window with table*

window which does not display the third solution. The table in Figure 2 illustrates that testing a few numerical values helps to discover an interval where the third solution is located (as seen in Figure 2). The graph of the difference of the functions (Figure 3), in general, makes it easier to find the zeroes. This graph can also help to build intuition as to why the fundamental theorem of algebra is true; showing that a cubic cannot have only two real solutions (each of multiplicity 1). This first example illustrates the importance of providing students with an informal introduction to the relative growth rate of different functions. It has been our experience that it takes only a few exercises, numerically comparing polynomials of different degrees and different families of continuous functions, for the students to get a sense of their relative growth. Without this exposure students tend to trust what they observe in the initial window displayed by the HHGT and miss hidden solutions. The second example (Figure 4) points to the need for knowing that the exponential function grows faster than most continuous functions in order to suspect the existence of the third solution. The graph of the difference of the two functions $f5(x) = f1(x) - f2(x)$ makes it easier to recognize that the solution to the inequality is approximately $(-0.095, 0.105) \cup (9, \infty)$, the part of the graph that lies below the *x*-axis.

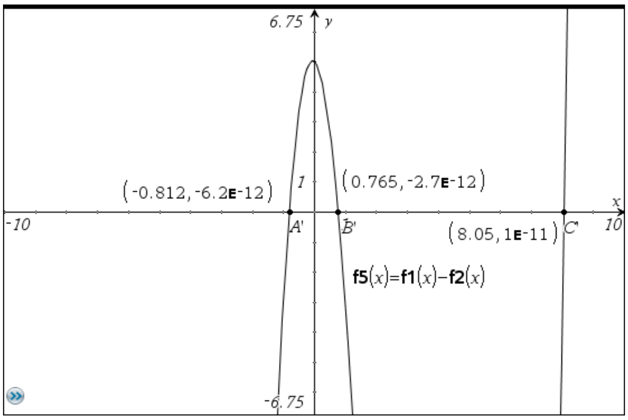


Figure 3: *Difference function for 1 a)*

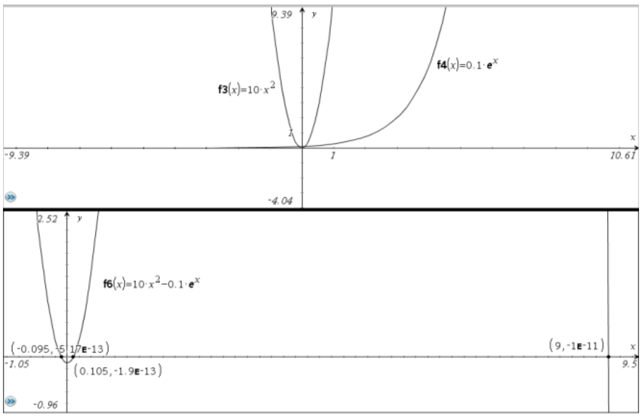


Figure 4: *Functions and their difference for 1 b)*

Example 2. Local and end behavior of continuous functions.

a) What is the local behavior of $f2(x) = \dfrac{3|x+2|}{2|1-\sqrt{x+3}|}$ when $x \to -2$ and b) $f1(x) = \dfrac{\sin(x - \pi/2)}{x - \pi/2}$ when $x \to \pi/2$? c) What is the end behavior of $f3(x) = \dfrac{x^3 - 3x + 2}{x - 6}$, as $x \to \pm\infty$?

This example addresses the study of the local and end behavior of a function numerically using sequences. Figure 5 shows a graph that illustrates how a calculator may seem not to recognize a discontinuity, in this case at *x=-2,* of the first function. This shows the importance of students not being overly reliant on calculators. However, we see also that by using a table, the student may easily analyze the behavior of the function when *x* approaches *-2* from either side. A faster, though more sophisticated approach, would be to evaluate the function at the values $-2 \pm 10^{-n}, n \in \mathbb{N}$, which are displayed in Figure 5, and using recursion in Figure 6. To avoid students becoming overly reliant on calculators, we need to carefully consider which algebraic skills are still required.

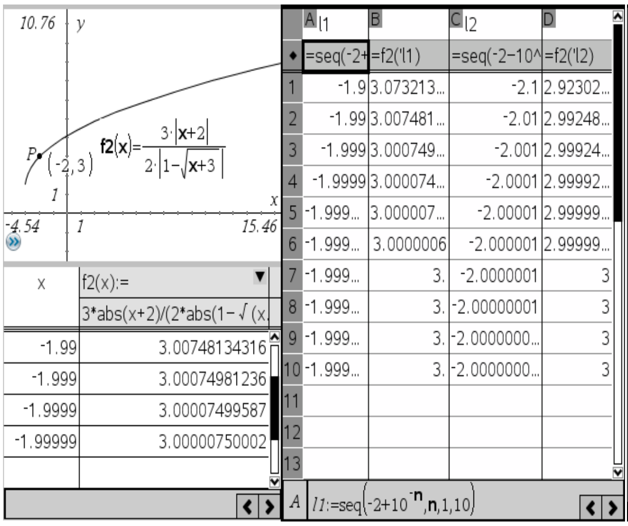


Figure 5: *Local behavior for2 a)*

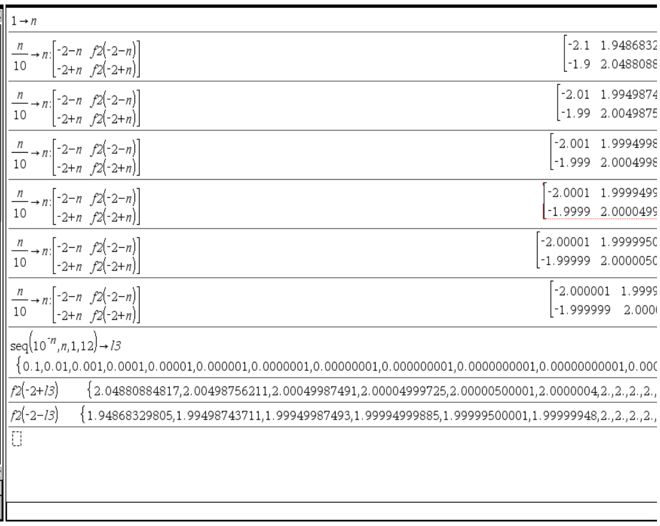


Figure 6: *Approaching -2 recursively*

In this example, the limit could be solved through algebraic manipulations that rationalize the denominator and show that this function is equivalent to $\frac{3}{2}\left(1+\sqrt{x+3}\right)$ when $x \neq -2$. But perhaps it is enough to have students understand why the function is discontinuous at –2 and that it’s graph will have a shape similar to the graph of $\frac{3}{2}\sqrt{x}$. In Figure 7 we see an example of a transcendental

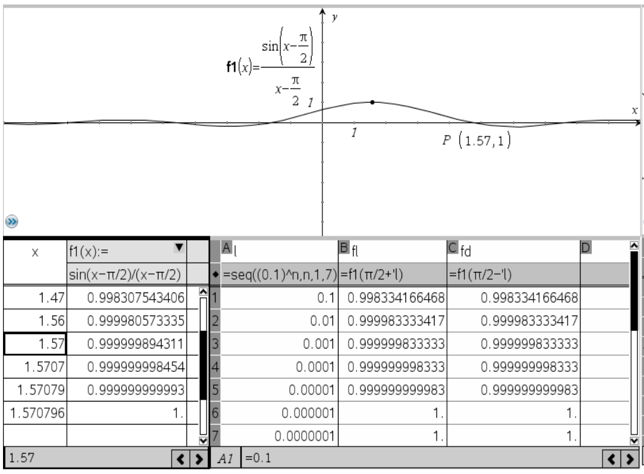


Figure 7: *Graph and table for 2 b)*

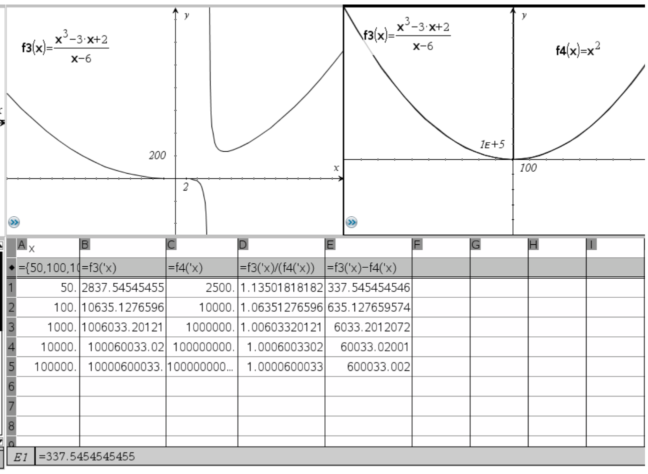


Figure 8: *Graph and table for 2 c)*

function typically considered in introductory calculus. In contrast with the algebraic methods traditionally used, the numerical approach works with any function and reinforces the idea of the behavior of a function when x gets arbitrarily close to a given value. As seen in Figure 8, the graphs contrast the local and the end behavior of the function. In the first graph of Figure 8 the vertical asymptote at –6 is apparent and in the second graph the end behavior of the function becomes indistinguishable from that of the parabola $y = x^2$ by taking a sufficiently large window, since the local behavior vanishes. The numerical values in the table illustrate how the quotient of both functions approaches *1* as *x* becomes increasingly large.

Example 3. Modeling and optimization.
a) Find the dimensions of the largest wood beam (in the shape of a rectangular prism) that can be obtained from a cylindrical tree with height *L* and radius of 4 decimeters. b) Find when the volume will be larger than 14*L* cubic decimeters.

Traditionally this problem is studied in the first course of differential calculus, but with the help of technology it can studied using different approaches at different levels before calculus. Since the volume of the beam is proportional to the area of the base, we need only to determine the rectangle of largest area that can be inscribed on a circle of radius 4 decimeters. Using dynamic geometry software (Figure 9) we have created a geometrical model and linked the variable's base and area

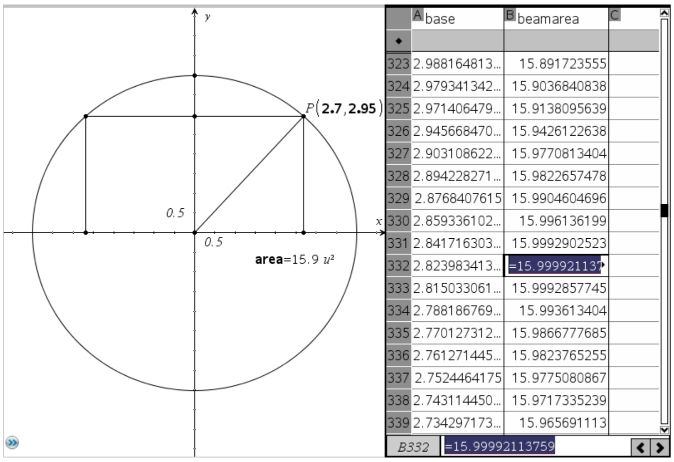


Figure 9: *Base area model and table for 3 a)*

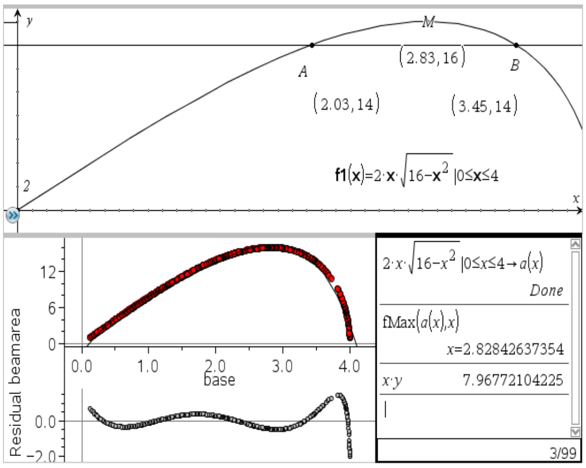


Figure 10: *Graph and residuals for 3 b)*

with the spreadsheet. Then by moving the point P along the circle, the changing values of these variables are collected. The largest value can then be simply read from the spreadsheet providing a good approximation. In Figure 10 we see that once a function modeling the area is obtained, the result sought corresponds to the maximum and can be obtained directly either from the graph or by using the operator that finds the maximum of the function. It also shows the points A and B obtained by intersecting *f1(x)* with *y=14*, which provides the interval (2.03, 3.45) of all base values that yield a volume larger than 14 (a question seldom asked in this type of problem). Finally, we have used a quartic polynomial to best fit the scatter plot of the points previously generated. The graph of residuals is included to show the error in the calculation of the maximum in this case.

These examples illustrate the potential that technology has to change how and what we teach. The approaches demonstrated to solve these types of problems are by no means comprehensive or even the best approaches. Depending on the students' backgrounds and the particular problem at hand, other approaches may indeed be more appropriate. For instance, there is a simple and elegant geometrical solution to the problem in example 3 which uses the fact that the rectangle of largest area inscribed in a circle is a square. However, the approaches demonstrated can be used to solve many problems and build intuition before introducing more advanced concepts. Technology allows a continuous interplay between graphical, numerical and analytical representations. This interplay can help students make more connections thereby deepening conceptual understanding: both of the traditional content and the new content. The new approaches and capabilities allow earlier access to some concepts and make a variety of relevant life examples obtainable.

4. Methodology

To determine whether teachers are taking advantage of technology integration as described in the previous two sections, we developed a test and a self assessment survey based on the previous list of major possible changes that HHGT facilitates form section 2.

The test contains questions on new content as well as traditional content because we believe that proper technology integration will lead to a deeper conceptual understanding of traditional content. We constructed the questions based on our years of experience with technology integration. This test was administered to in- and preservice mathematics teachers in the spring semester of 2008 to get a sense of how well prepared they were for integrating technology in topics foundational to calculus. The test questions were categorized according to the following learning outcomes that were based on the criteria from section 2:

1. Understanding attributes of functions such as their complete graphs, local and end behavior, one-to-oneness, inverse behavior, continuity, intercepts, domain, range, etc.
2. Solving equations from graphs, algebraic expressions, etc.
3. Solving inequalities from graphs, algebraic expressions, etc.
4. Setting up applications
5. Transformations
6. Modeling data via regression
7. Relative growth between families of continuous functions
8. Making connections between representations such as graphs, equations and inequalities, as well as between abstract to concrete information.

Questions were also categorized as follows:

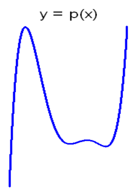
9. Any test problems involving polynomial functions.
10. Any test problems involving rational functions.
11. Any test problems involving exponential and logarithmic functions.
12. Any test problems involving trigonometric functions.
13. Any test problems involving functions other than those previously listed, such as radicals, absolute values, etc.

Many of these categories overlap. For example, a question that asked about the shape of a graph of a polynomial function would fall under both the first (1) and ninth (9) learning outcomes. A sample of the test questions is included in Table 1 along with the learning outcomes used to categorize them.

Table 1: *Sample of test questions*

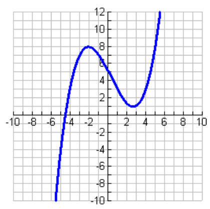
1. The minimum possible degree of the polynomial $p(x)$ depicted on the right is

A. 6 B. 5 C. 4 D. 3 E. Can not be determined from graph

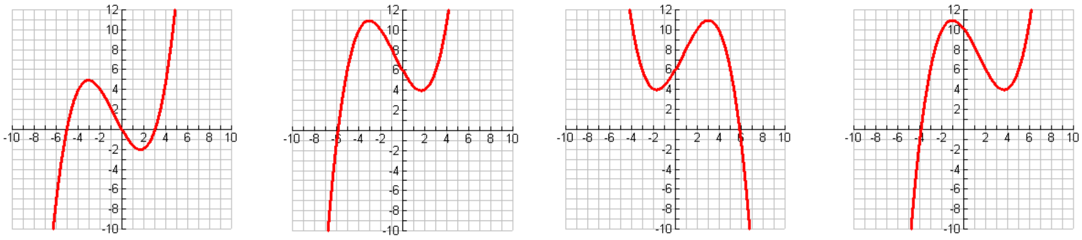


This question falls under the categories of understanding the local and end behavior of functions (1), the notion of the complete graph of a function (1), making connections between graphs and equations (8) and polynomials (9).

2. The graph to the right shows the curve of $y = f(x)$. Which of the four graphs below shows the curve $y = 3 + f(x-1)$?



This question falls under the categories of transformations (5), connections between analytic and graphical representations (8) and polynomials (9).



None of these

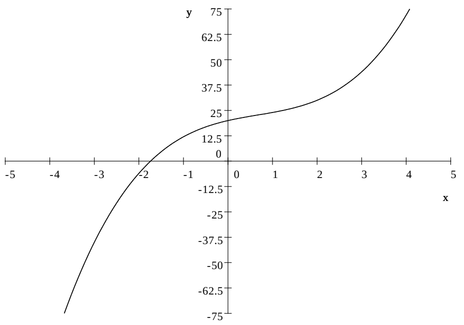
A B C D E

3. Find the domain and range of the function $h(x) = x - \sqrt{x}$.

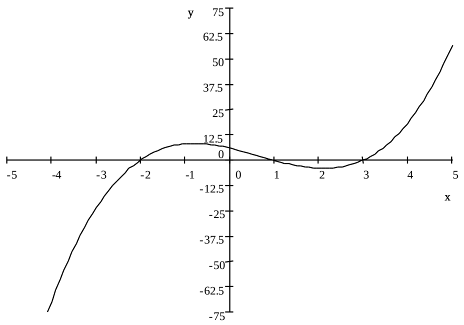
This question falls under the categories of understanding the attributes of functions (1) and radical functions (13).

4. The sum and difference of two functions, $f(x)$ and $g(x)$, are provided below. Determine all values of x in the interval $(-5, 5)$ that satisfy $f(x) = g(x)$.

This question falls under the categories of solving equations from graphs (2) and connections between graphical and abstract representations (8).



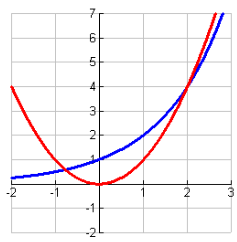
$f(x) + g(x)$



$f(x) - g(x)$

5. Below, <u>incomplete</u> graphs of the functions $y = 2^x$ and $y = x^2$ are provided. <u>How many</u> solutions are there to $2^x = x^2$?

This question falls under the categories of understanding the end behavior of functions (1), the notion of the complete graph of a function (1), solving equations from graphs (2), the relative growth of functions (7), polynomials (9) and exponential functions (11).

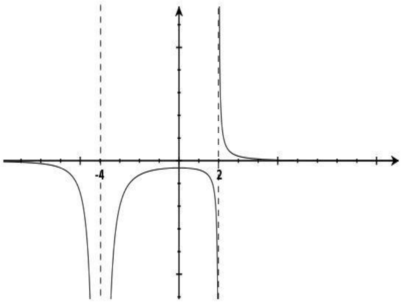


6. Approximate, to the nearest integer, the value of $f(x) = \dfrac{3x^2 + 1}{1 - x^2}$ for $x = 4^{100}$.

This question falls under the categories of understanding the end behavior of functions (1) and rational functions (10).

7. Consider the graph of the rational function r(x) depicted. Determine a function that generates the graph of r(x).

This question falls under the categories of understanding the attributes of functions (1), making connections between graphs and equations (8) and rational functions (10).



8. The table below contains numerical values for the continuous $f(x)$ in the interval (-1.5, 1.5). What properties of the graph of $f(x)$ can you determine from the table? (Remark: You can answer with a value and/or with an interval of values.)

x	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.1	0.3	0.6	0.8	1	1.2	1.4
f(x)	-.29	-.06	0	-.05	-.16	-.29	-.38	-.4	-.36	-.17	-.51	1.3	2.4	3.9	5.8

Zero(s):______________________; Local extrema: $\begin{cases} \text{max}: \_\_\_\_\_\_\_\_\_\_\_\_ \\ \text{min}: \_\_\_\_\_\_\_\_\_\_\_\_ \end{cases}$

This question falls under the categories of understanding the attributes of continuity, intercepts, and the complete graph of functions(1) and making connections between tables and abstract concepts such as continuity and local behavior (8).

The questions on the self assessment survey are located in Tables 4 and 5 of the next section. These surveys contained questions on experience with different forms of technology and experience with how technology could be integrated when teaching content prerequisite to calculus. The results on the surveys were reflected in the test results and they also provided explicit details on the lack of preparation received by the in- and preservice teachers.

5. Results

The test was administered to three intact groups of 13, 15, and 18 preservice secondary mathematic teachers from three universities in the Midwest of the USA. In two of the universities participants were registered in a course. The third university was between terms, so participants were found by advertising the test among secondary mathematics majors and raffling a graphing calculator among those taking the test. Participants were juniors and seniors, and had therefore completed the calculus sequence.

It is important to remark that the participants came from three recognized and well-respected universities, two public and one private. Moreover, the mean grade point average (GPA) of all participants was 3.6 out of 4.0, while their mean GPA in mathematics courses was 3.5; grades that would seem to indicate a well-prepared group of students capable of coping with a set of questions at the precalculus level.

The same test was also given in the spring semester, during a workshop, to a group of 74 secondary and middle school inservice teachers from 43 high schools and 6 middle schools. These teachers represented 40 school districts from the same geographical area. The average number of years of experience was 12. In addition, 56 of the teachers had completed master degrees in education, 4 of the teachers had master's degrees in mathematics, and another 2 had master's degrees in business.

Fifty-three percent of the preservice teachers and 58% of the inservice teachers were female. Descriptive statistics with the test results for in- and preservice teachers are included in Table 2.

Table 2: *Descriptive statistics for inservice and preservice teachers*

	74 Inservice Teachers			46 Preservice Teachers		
	Mean (%)	Std. Dev. (%)	Std. Error Mean (%)	Mean (%)	Std. Dev. (%)	Std. Error Mean (%)
Test	54.3	23.9	2.8	46.3	18.5	2.7
1. Attribute	59.4	22.4	2.6	51.1	18.3	2.7
Complete	65.3	26.0	3.0	54.8	24.4	3.6
Local/End	65.5	26.7	3.1	60.6	23.1	3.4
One-to-One	67.6	47.1	5.5	73.9	44.4	6.5
Inverse	33.8	47.6	5.5	17.4	38.3	5.7
Continuity/Intercepts	27.0	44.7	5.2	27.2	43.1	6.4
Domain/Range	58.8	28.8	3.3	55.7	21.7	3.2
2. Equations	41.0	32.7	3.8	30.0	29.3	4.3
Equation to Graph	46.6	38.2	4.4	35.9	35.1	5.2
Equation to Equation	35.5	37.5	4.4	24.1	31.2	4.6
3. Inequalities	54.4	35.1	4.1	40.0	29.5	4.3
Inequality to Equality	55.2	40.8	4.7	39.1	34.6	5.1
4. Applications	52.1	32.4	3.8	30.7	28.3	4.2
5. Transformations	55.6	29.2	3.4	42.2	24.5	3.6
6. Regression	29.7	42.9	5.0	9.8	29.1	4.3
7. Relative Growth	39.9	35.1	4.1	33.7	36.6	5.4
8. Connections	48.4	26.6	3.1	41.4	21.4	3.2
Graph to Equation	52.1	28.3	3.3	46.7	21.8	3.2
Equation to Graph	54.4	36.9	4.3	48.6	39.4	5.8
Abstract to Other	46.6	33.7	3.9	39.1	31.1	4.6
9. Polynomial	53.8	28.1	3.3	43.3	22.4	3.3
10. Rational	56.2	25.2	2.9	44.2	20.2	3.0
11. Exponential or Log	56.1	37.2	4.3	52.7	32.2	4.7
12. Trigonometric	53.4	37.3	4.3	41.8	36.1	5.3
13. Other Functions	50.2	38.1	4.4	38.4	34.9	5.2

The mean scores on the tests for the inservice and preservice teachers were 54% and 46%, respectively. These low averages indicate a need for improvement on the topics that were on this test. In fact, none of the thirteen categories scored a mean over 60%. However, some categories fared worse than others. The categories that need the most improvement are Regression (mean scores ranged from 10% – 30%), Solving Equations (mean scores ranged from 30% – 41%), and Relative Growth (mean scores ranged from 34% – 40%). These are followed by Applications (mean scores ranged from 31% – 52%), Other Types of Functions (mean scores ranged from 38% – 50%), and Connections (mean scores ranged from 41% – 48%). The subcategories that need the most improvement are Attribute questions involving Inverses (mean scores ranged from 17% – 34%), Attribute questions involving Continuity and Intercepts (27% for both groups), Solving Equations from Equations (mean scores ranged from 24% – 36%), Solving Equations from Graphs (mean scores ranged from 36% – 47%), and Connections from Abstract Information to Concrete Information (mean scores ranged from 39% – 47%). To reiterate, all of the categories need improvement, some more than others.

It was not surprising that the inservice teachers scored a higher average on the tests because as many teachers discover (ourselves included), teaching a concept deepens understanding of that concept. To investigate further, independent sample t tests were performed on the test results. The outcomes are in Table 3. Levene's test for equality of variances was used to determine whether a

Table 3: *Independent sample t tests*

	Equal var. assumed (EVA) Equal var. not assumed (EVNA)	Levene's Test for Equality of Variances		t test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Differ.	Std. Error Differ.	95% Confidence Interval of the Difference	
									Lower	Upper
Test	EVNA	5.274	0.023	2.054	112	0.042	0.080	0.039	0.003	0.157
1. Attribute	EVA	1.931	0.167	2.122	118	0.036	0.084	0.039	0.006	0.161
Complete	EVA	0.123	0.727	2.193	118	0.030	0.105	0.048	0.010	0.199
Local/End	EVA	0.518	0.473	1.037	118	0.302	0.049	0.048	-0.045	0.144
One-to-One	EVA	2.301	0.132	-0.733	118	0.465	-0.063	0.087	-0.235	0.108
Inverse	EVNA	18.841	0.000	2.072	110	0.041	0.164	0.079	0.007	0.321
Continuity/Intercepts	EVA	0.185	0.668	-0.018	118	0.986	-0.001	0.083	-0.165	0.162
Domain/Range	EVNA	5.776	0.018	0.665	114	0.508	0.031	0.046	-0.061	0.123
2. Equations	EVA	1.173	0.281	1.872	118	0.064	0.111	0.059	-0.006	0.228
Equation to Graph	EVA	0.108	0.743	1.546	118	0.125	0.108	0.070	-0.030	0.245
Equation to Equation	EVNA	4.387	0.038	1.796	108	0.075	0.114	0.063	-0.012	0.239
3. Inequalities	EVNA	4.883	0.029	2.418	108	0.017	0.144	0.060	0.026	0.262
Inequality to Equality	EVNA	5.937	0.016	2.302	107	0.023	0.160	0.070	0.022	0.299
4. Applications	EVA	3.789	0.054	3.699	118	0.000	0.214	0.058	0.100	0.329
5. Transformations	EVNA	2.618	0.108	2.710	108	0.008	0.134	0.050	0.036	0.233
6. Regression	EVNA	30.732	0.000	3.031	117	0.003	0.199	0.066	0.069	0.330
7. Relative Growth	EVA	0.830	0.364	0.921	118	0.359	0.062	0.067	-0.071	0.194
8. Connections	EVA	3.807	0.053	1.521	118	0.131	0.071	0.046	-0.021	0.163
Graph to Equation	EVNA	5.643	0.019	1.165	113	0.246	0.054	0.046	-0.038	0.145
Equation to Graph	EVA	0.417	0.520	0.822	118	0.413	0.058	0.071	-0.082	0.199
Abstract to Other	EVA	0.298	0.586	1.218	118	0.225	0.075	0.061	-0.047	0.197
9. Polynomial	EVNA	4.531	0.035	2.263	111	0.026	0.105	0.046	0.013	0.197
10. Rational	EVA	3.308	0.071	2.714	118	0.008	0.119	0.044	0.032	0.206
11. Exponential or Log	EVA	3.878	0.051	0.507	118	0.613	0.034	0.066	-0.098	0.165
12. Trigonometric	EVA	0.446	0.505	1.674	118	0.097	0.116	0.069	-0.021	0.253
13. Other Functions	EVA	2.027	0.157	1.706	118	0.091	0.118	0.069	-0.019	0.255

pooled- or separate-variance t test should be employed. Equality of variances was assumed if the F statistic had a significance level of at least .05. In this case the pooled-variance t test was used. The mean score on the test for the inservice teachers (M = 54%, SD = 0.24) was significantly higher than the mean score on the test for the preservice teachers (M = 46%, SD = 0.19), t(112) = 2.05, p = .04 (two-tailed). In seven of the thirteen categories, the t tests showed that the inservice teachers had significantly higher means (Attributes, Inequalities, Applications, Transformations, Regression, Polynomial functions and Rational functions). There was not a significant difference in the other six categories. Because the inservice teachers performed better on the test in all 13 categories and it appeared that as the preservice teachers' scores increased on a question, so too did the inservice teachers' scores, there was a good possibility that the average question scores between the two groups might be positively correlated. That was the case. Question scores were positively correlated (0.898), p < .001 (two-tailed). This again supports the hypothesis that the experience of the inservice teachers deepened their understanding of concepts.

The preservice and inservice teachers responded to survey questions that related integrating technology with a variety of content. The focus for the preservice teachers was their knowledge of how to do this and the focus for the inservice teachers was how often they taught this way. The results are in Table 4**.** The average ratings mainly occurred inside the "very little" and "some"

Table 4: *Self-ratings on technology used in teaching or learning content*

The following Likert scale was used on these questions: 1=Not at all 2=very little 3=some 4=often 5=Continuously Preservice: Rate your knowledge integrating technology into each of the following areas. Inservice: How often (if at all) do you teach each topic via technology?	Preservice Averages	Inservice Averages
1. The use of nontraditional tools such as lists, sequences, recursion to solve different problems?	2.69	2.38
2. The consistent interplay of these 3 representations	3.20	2.85
3. Calculating the range of functions using extrema?	2.81	2.30
4. Calculating intervals where a function is increasing or decreasing?	3.39	2.59
5. The local behavior of functions via approximations?	3.02	2.33
6. The global behavior of functions via approximations?	2.74	2.20
7. Optimization problems for each family of functions besides quadratics?	2.54	2.10
8. Solving transcendental equations graphically?	2.48	2.09
9. Solving transcendental equations numerically?	2.62	2.11
10. Solving transcendental inequalities?	2.52	1.90
11. Questions such as "when will the answer be at least (at most) some number?" rather than just asking "when will the answer be some number."	3.17	2.32
12. Family of functions as coming from a root, via transformations?	2.66	2.32
13. Modeling real data using nonlinear regression for each family of continuous functions?	2.42	2.09
14. Matrix applications (Networks, Markov, Transf…)	2.64	1.86
15. Recursion	2.50	1.73
Average Score	2.76	2.21

categories. These low ratings are consistent with the low test scores achieved by the respondents. These low ratings also indicate that the respondents were aware of their weaknesses in these areas. It is interesting to note that the preservice teachers gave themselves higher ratings than the inservice teachers on every question. Also, the average survey question scores between the two groups were positively correlated (0.786), p = .001 (two-tailed). This might indicate that a deficiency in knowledge of integrating technology with content before becoming a teacher, leads to a deficiency in using technology when teaching content, and that after teaching for a while the inservice teachers gave themselves lower ratings because they were aware of the fact that they do not integrate technology thoroughly. However, it could be that the preservice teachers have more comfort with technology due to more exposure in their preservice period and perhaps this will lead to them integrating technology more thoroughly.

The pre-service and in-service teachers also responded to survey questions where they rated the usage of different types of technology in the classroom. The focus for the pre-service teachers was usage in the classes that they had taken and the focus for the inservice teachers was usage in the classes that they had taught. The results are in Table 5. The inservice teachers gave themselves the

Table 5: *Self-ratings on types of technology used in classroom*

	The following Likert scale was used on these questions: 1=Not at all 2=very little 3=some 4=often 5=Continuously	Preservice	Inservice
I	Preservice: Do <u>the classes you have taken</u> integrate the use of technology? Inservice: Do you integrate the use of technology in the teaching/learning of mathematics?	2.92	3.38
II	<u>Rank</u> how often those classes integrated each technology.		
	a) Scientific Calculators	1.95	3.53
	b) Graphing calculators	3.07	3.47
	c) Math Software, which one(s)?	2.00	1.99
	d) Dynamic Geometry Software (Cabri, Geometers Sketchpad, Other:	2.91	1.67
	e) Other kind of technology, which one(s):	1.83	2.38
Average Score		2.45	2.73

highest ratings in calculator usage. The average ratings for calculator usage occurred between the "some" and "often" categories. Combining these responses with the test results and previous self-ratings, it seems likely that even though the inservice teachers are using calculators regularly in their classrooms, they are using them with a limited scope. The ratings from the preservice teachers were lower on average and had a smaller range that mainly occurred between the "very little" and "some" categories. They gave their highest ratings for graphing calculator and dynamic geometry software usage. These ratings support the test results and previous self-ratings.

The limitations teachers have with HHGT do not appear to be exclusive to the precalculus topics. A different group of 69 inservice teachers were tested and surveyed in 2009 [2] on their knowledge of integrating technology for teaching content from probability and statistics, data analysis, matrix applications and discrete mathematics. The mean scores on the tests were 38% (35% for probability and statistics, 39% for data analysis, 27% for matrix applications and 52% for discrete mathematics) [2]. These extremely low scores indicate weakness in this content beyond using technology. On the survey questions, the inservice teachers rated themselves with an average of 2.71 for using calculators in these topics (where 1 = None and 5 = High Level) [2]. When asked how often they used technology to teach these topics, all of the averages fell below "some", which was a 3, with an overall average of 2.1 [2]. These test scores and self ratings were lower than for the precalculus content group. This might be due to the fact that these topics are not required in the curriculum and thus fewer teachers would instruct this material.

6. Conclusions

The performance of preservice and inservice teachers on the test questions and learning outcomes seem to indicate a lack of exposure to these topics in their preparation to become teachers. This is corroborated by the self-evaluation of both groups on the chosen topics. Judging by these results the answer to our initial question seems to be that we are not taking full advantage of the range of capabilities that HHGT offers in order to provide precalculus students with the best possible preparation for calculus. It seems that neither the mathematics courses nor the methods courses that these participants had taken prepared them to answer the representative set of questions using HHGT. It may be the case that some of the topics we chose are not as relevant as we would like to think. Even if this is the case, we need to ask why our inservice and preservice teachers are not prepared to deal with the questions addressing the remaining topics. Are the mathematics educators and mathematics faculty who are teaching these students knowledgeable about the new approaches and concepts that HHGT makes possible? Is the information about the educational possibilities of HHGT and the research results, that point to the positive impact of this technology, reaching this faculty? The regular use of HHGT by inservice teachers in their classrooms does not seem to have contributed to an improvement in the scope of what they teach. Therefore, new and more advanced technologies will probably not help either, unless we improve the preparation of preservice teachers on the proper integration of technology.

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